String-like structures in complex Kerr geometry: Calabi-Yau twofold from the Kerr theorem

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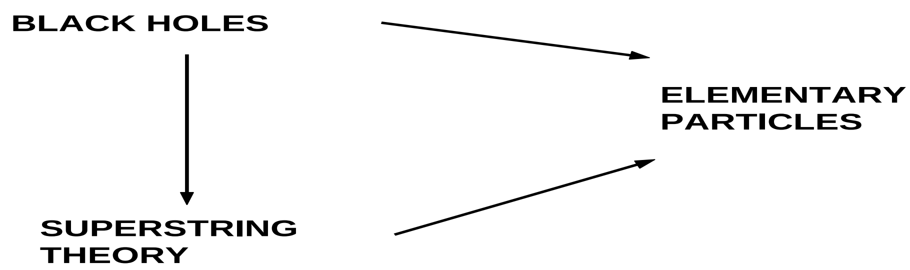
based on:

A.B., *String-like structures in the four-dimensional Kerr geometry: ..* [arXiv: arXiv:1211.6021];

BLACK HOLES - STRINGS - PARTICLES It is commonly recognized now that black holes are related with elementary particles and string theory ['t Hooft (1990), A.Salam and J. Strathdee (1976), Witten (1992), C.F.E. Holzhey and F. Wilczek (1992), A. Sen (1995), at al.].



FUNDAMENTAL STRINGS are soliton like solutions to low-energy string theory.
Some solutions to Einstein's eqs. are exact solutions to effective string theory. PP-WAVES, Horowitz & Steif (PRL 1990), A. Tseytlin (PRD 1993).
Strings as Solitons & Black Holes as Strings. Dabholkar at.al (1990-1995).

FUNDAMENTAL CHARGED HETEROTIC STRING solution, [A. Sen (NPB 1992-1993)]

Traveling waves as modes of string excitations, D.Garfinkel (PRD 1992).

TWISTORS: Kerr-Schild $\Leftrightarrow$ TWISTOR-STRING (Nair, Witten), Twistors in N=2 critical string (Ooguri-Vafa 1991)
Kerr-Newman (KN) solution: gyromagnetic ratio $g = 2$ $\Rightarrow$ **gravitationalbackground of electron!** [Carter, Israel, Debney&Kerr&Schild, Lopez, AB.]

Spinning particles ! $\Rightarrow$ (spin/mass) ratio, $J/m > 10^{20}$ (units $G = \hbar = c = 1$) $\Rightarrow a = J/m >> m$, and **black hole horizons disappear**: over-rotating Kerr geometry!

NAKED KERR's SINGULAR RING. – the spacetime is a singular, twosheeted and has a closed timelike curves!

METRIC SHOULD BE REGULATED TO A FLAT SPACETIME NEAR THE CORE - 4d analog of the "Repulson" and enhancon model in M-theory [Townsend, Sen, Polchinski, Johnson&Järv et al. (1995-2000)]

Kerr-Sen solution to low energy string theory: The Kerr solution with axion and dilaton, A. Sen (PRL 1992). Regulated Kerr singular ring may be interpreted as *a closed fundamental string.* [Sen (1992), AB (1995)]

The Kerr SINGULAR RING is a 'closed' heterotic string. The field around Kerr-Sen solution to low energy string theory is similar to the Sen solution for HETEROTIC STRING. AB (PRD 1995) (Lightlike circular currents.)

*Aim of this treatment is an **open and complex** string which emerges in the complex Kerr geometry.*

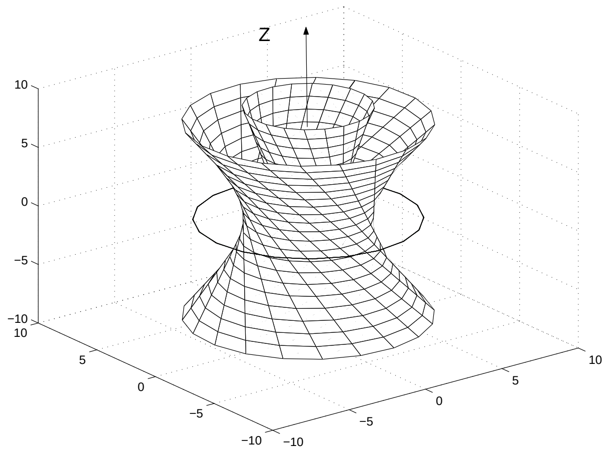
Structure of the REAL Kerr-Newman solution: Metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \quad H = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}, \tag{1}$$

and electromagnetic (EM) vector potential is

$$A^\mu_{KN} = Re\frac{e}{r + ia\cos\theta}k^\mu. \tag{2}$$

Gravitational and EM fields are concentrated near the Kerr singular ring.



The Kerr ring forms a branch line of space. The KN geometry is TWOSHEETED! Vector field $k_\mu(x)$ is tangent to Principal Null Congruence (PNC),

$$k_\mu dx^\mu = P^{-1}(du + \bar{Y}d\zeta + Yd\bar{\zeta} - Y\bar{Y}dv), \quad Y(x) = e^{i\phi}\tan\frac{\theta}{2}, \tag{3}$$

where $Y(x)$ is projective angular coordinate, and

$$\zeta = (x + iy)/\sqrt{2},\ \bar{\zeta} = (x - iy)/\sqrt{2},\ u = (z - t)/\sqrt{2},\ v = (z + t)/\sqrt{2}$$

are the null Cartesian coordinates.

Kerr congruence is controlled by the

KERR THEOREM:

The geodesic and shear-free Principal null congruences (type D metrics) are determined by holomorphic function $Y(x)$ which is analytic solution of the equation

$$F(T^a) = 0 \ , \tag{4}$$

where F is an arbitrary analytic function of the ***projective twistor coordinates***

$$T^a = \{Y, \quad \zeta - Yv, \quad u + Y\bar{\zeta}\}. \tag{5}$$

Complex radial distance $\tilde{r} \equiv r + ia\cos\theta = -dF/dY.$

The Kerr theorem is a practical tool for obtaining exact solutions:

$$F(T^a) = 0 \Rightarrow F(Y, x^\mu) = 0 \Rightarrow Y(x^\mu) \Rightarrow k^\mu(x)$$

For the Kerr-Newman solution function F is quadratic in Y, which yields TWO roots $Y^\pm(x) \Rightarrow$ two different congruences at the same background M^4!
Functions $F(T^a)$ of higher degrees in Y correspond to multi-particle solutions, [AB (2006)].

Kerr singular ring $r + ia\cos\theta = 0$ is a branch line of space on two sheets: "negative $(-)$" and "positive $(+)$", where the fields change their directions. In particular,

$$k^{\mu(+)} \neq k^{\mu(-)} \quad \Rightarrow \qquad g_{\mu\nu}^{(+)} \neq g_{\mu\nu}^{(-)}. \tag{6}$$

Twosheeted mystery created the problem of source of the KN solution.

Kerr's oblate spheroidal coordinates $x + iy = (r + ia)e^{i\phi}\sin\theta, \quad z = r\cos\theta$, cover spacetime twice: disk $r = 0$ separates the 'out'-sheet $r > 0$, from the 'in'-sheet $r < 0$.

(a) ***Closed fundamental string:*** AB 1974, Gravitational strings: D.Ivanenko & AB 1975, W.Israel 1977, Fundamental solitonic string solution to low energy string theory, G. Horowitz and A.Steif (1990), A. Sen (1992 -1995), A. Dabholkar et al.(1995), AB(1995-2011).

(b) ***Relativistically rotating disk. Truncation of the Kerr negative sheet*** H.Kerres (1967), W.Israel (1969), Hamity, I.Tiomno (1973).

(c) ***Relativistically rotating membrane (bubble),*** C.López (1983) .

(d) ***Gravitating soliton: supersymmetric vacuum bubble bounded by a closed string***, AB (2010).

Close parallelism with the problem of repulson singularity in superstring/M-theory unification, excise of singularity and the model of enhancon.

(e) ***Complex KN source as a COMPLEX STRING***, AB (1993-2012).

GRAVITATING SOLITON (AB, 2010) – chiral Higgs model. Supersymmetric phase transition from external KN solution to a 'false vacuum' bubble bounded by the domain wall M2-brane.

Perspective goal – description of the Weinberg-Salam model.

Peculiarities of the KN soliton model:

(i) the Kerr ring is regularized, forming a closed relativistically rotating string of the Compton radius r_c on the border of disklike membrane,

(ii) the KN electromagnetic potential forms a quantized loop $\oint eA_\varphi d\varphi = -4\pi ma$, which results in quantization of the soliton spin, $J = ma = n\hbar/2, \ n = 1, 2, 3, ...,$

(iii) the Higgs condensate forms a coherent vacuum state oscillating with the frequency $\omega = 2m$ – oscillon,

Complex Structure of the Kerr geometry.

Kerr's complex radial distance $\tilde{r} \equiv r + ia\cos\theta = x^2 + y^2 + (z+ia)^2$, in Cartesian coordinates shows that it is a **complex shift of the real one.**

Complex shift of the Coulomb solution $\Phi = Re(q/\sigma)$ Appel solution 1887!

r and θ turn into Kerr's oblate spheroidal coordinates.

There is exact correspondence between Appel's complex shift and Kerr-Schild geometry. The Kerr-Newman solution is generated by a complex source, positioned in complex region! Newman's retarded-time construction (1973).

Complex Kerr-Schild geometry.

Complex light cones *with the vertexes on the* complex world-line $x_0^\mu \in CM^4$: $(x_\mu - x_{0\mu})(x^\mu - x_0^\mu) = 0$, are splits into two families of the "left" and "right" complex null planes: $x_L^\mu = x_0^\mu(\tau) + \alpha e^{1\mu} + \beta e^{3\mu}$ e^1 and e^3, and $x_R^\mu = x_0^\mu(\tau) + \alpha e^{2\mu} + \beta e^{3\mu}$, spanned by the null tetrad e^a, $(e^a)^2 = 0$.

Twistors are created by complex shift!

The Kerr congruence arises from real slices of the family of the "left" null planes ($Y = const.$) of the complex light cones whose vertices lie at a complex world-line $x_0(\tau)$.

Complex string as source of the Kerr geometry. AB [gr-qc/9303003, 1203.4210]. **Kerr's geometry is created by a mysterious "particle" propagating along a** *complex world-line* **(CWL)** $x_0^\mu(\tau)$ **parametrized by complex time** $\tau = t + i\sigma$**.**

It forms a world-sheet. [Earlier discussion of the complex world-line as a string by Oogury and Vafa (1991).]

The corresponding **"hyperbolic string" equation** $\partial_\tau \partial_{\bar\tau} x_0(t,\sigma) = 0$, has the **general solution** $x_0(t,\sigma) = x_L(\tau) + x_R(\bar\tau)$ as a sum of the analytic and anti-analytic modes $x_L(\tau), \quad x_R(\bar\tau)$, which are not necessarily complex conjugate. The complex retarded-time parameters τ and $\bar\tau$. Four different roots for the Left and Right complex retarded-advanced times

$$\tau_L^\mp = t \mp (r_L + ia\cos\theta_L) \tag{7}$$

$$\tau_R^\mp = t \pm (r_R + ia\cos\theta_R). \tag{8}$$

The real slice condition determines relation $\sigma = a\cos\theta$ with null directions of the Kerr congruence $\theta \in [0,\pi]$, which puts restriction $\sigma \in [-a,a]$ indicating that *the complex string is open*, and its endpoints $\sigma = \pm a$ may be associated with the Chan-Paton charges of a quark-antiquark pair.

Boundary conditions require orientifold structure: the open string string is formed as **closed and folded**.

The complex conjugate world-lines, $X_L(\tau_L)$ and $X_R(\tau_R)$.

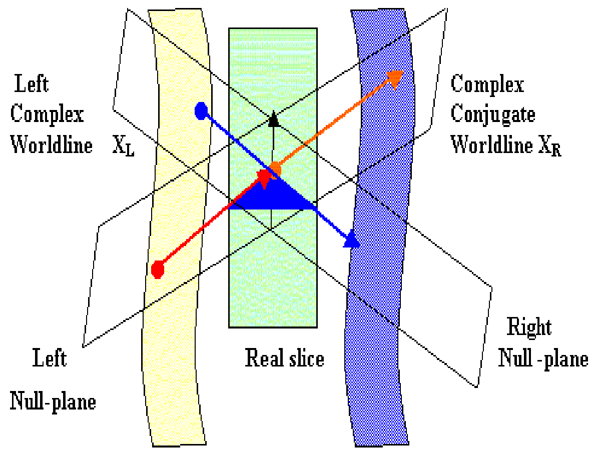


Figure 1: **Complex light cone at a real point x. The adjoined to congruence Left and Right complex null planes. Four roots: X_L^{adv}, X_L^{ret} and X_R^{adv}, X_R^{ret} which are related by crossing symmetry.**

Kerr theorem $\Rightarrow$ twoseetedness of the Kerr geometry.
For Kerr solution function $F(T^A)$ is quadratic in Y. It is a ***quadric*** in twistorial CP^3, $F = A(x^\mu)Y^2 + B(x^\mu)Y + C(x^\mu)$. Solution $Y^\pm(\mathrm{x}) = (-\mathrm{B} \mp \tilde{\mathrm{r}})/2\mathrm{A}$, where the complex radial distance $\tilde{r} = -(B^2 - 4AC)^{1/2}$.
Left and Right complex structures form a **wordsheet orientifold** of the complex string.
$\Omega = Compl.\ Conj. + Revers\ of\ radial\ coordinate.$
Antipodal map: $Y^+ \to -1/\bar{Y}^-$. Orientifolding of the retarded and advanced fields and the Kerr congruence Y^+ and Y^- : $\Omega + Antipodal\ map.$

Kerr theorem for multi-particle KS space-times.

Selecting an isolated i-th particle with parameters q_i, one can obtain the roots $Y_i^{\pm}(x)$ of the equation $F_i(Y|q_i) = 0$ and express F_i in the form

$$F_i(Y) = A_i(x)(Y - Y_i^+)(Y - Y_i^-). \tag{9}$$

Then, the $(+)$ or $(-)$ root $Y_i^{\pm}(x)$ determines congruence $k_\mu^{(i)}(x)$ and consequently, the Kerr-Schild metric $g_{\mu\nu}^{(i)} = \eta_{\mu\nu} + 2h^{(i)} k_\mu^{(i)} k_\nu^{(i)}$.

For a system of k particles we form the function F as a product of the known blocks $F_i(Y)$,

$$F(Y) \equiv \prod_{i=1}^{k} F_i(Y). \tag{10}$$

Solution of the equation $F = 0$ acquires $2k$ roots $Y_i^{\pm}$, and the twistorial space turns out to be multi-sheeted.

The twistorial structure on the i-th $(+)$ or $(-)$ sheet is determined by the equation $F_i = 0$ and does not depend on the other functions $F_j, \quad j \neq i$. Therefore, the particle i does not feel the twistorial structures of other particles. Similar, the condition for singular lines $F = 0, \ d_Y F = 0$ splits into k independent relations

$$F_i = 0, \quad \prod_{l \neq i}^{k} F_l d_Y F_i = 0. \tag{11}$$

The number of surrounding particles and number of blocks in the generating function F may be assumed countable. In this case the multi-sheeted twistorial space-time will possess the properties of the multi-particle Fock space.

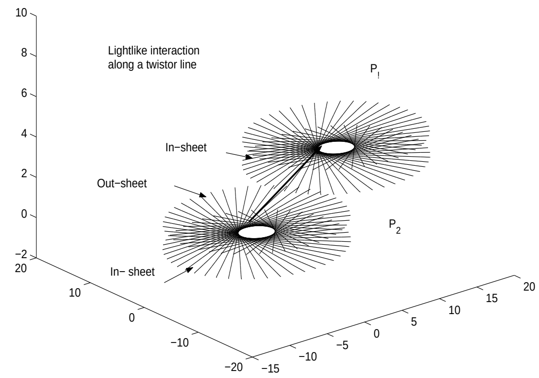


Figure 2: The lightlike interaction of two sources occurs via a common twistor line connecting out-sheet of one source to in-sheet of another.

The Left and Right structures by excitations should be considered as independent and generated by different KN sources, which corresponds to two-particle KN system with *quadratic* generating functions of the Kerr theorem $F_1(T)$ and $F_2(T)$, determined on the projective twistor space CP^3.

Kerr Theorem $\Rightarrow$ the orientifold twistor system is to be described by the generating function $F_{12}(T) = F_1(T) \cdot F_2(T)$. The corresponding equation

$$F_{12}(T) = F_1(T) \cdot F_2(T) = 0,$$

is $QUARTIC$ on the projective twistor space, and therefore the complex string forms a *Calabi-Yau twofold (K3) embedded in the projective twistor space*.

CONCLUSION:

Striking parallelism with superstring theory. Is it accidental, or there is inherent relationships with superstring theory? Too many coincidences!

In many respects the Kerr-Schild gravity resembles the twistor-string theory (Nair, Witten) which is also four-dimensional and based on twistors, which determines its relationship with particle physics.

The complex Kerr string has mach in common with the N=2 critical superstring, which is also related with twistors (Ooguri-Vafa) and has the real critical dimension four. Signature of the N=2 string is (2,2) or (4,0), which was principal obstacle for embedding in the Lorentzian space-times.

However, embedding of the N=2 string in the complexified Kerr geometry is trivial task. It is simple the Newman's Complex World Line. The transfer to supersymmetry corresponds to a super-world-line, in which the Appel complex shift is replaced by a super-shift.

We suppose that stringlike structures of the real and complex Kerr geometry are not simply analogues, but reflect the underlying dynamics of the N=2 superstring.

THANK YOU FOR ATTENTION!